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THE TRANSVERSE COLLECTIVE EFFECTS IN THE MAIN RING

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The Situation

Vertical and horizontal feedback systems have been installed in the main ring to damp transverse coherent motion of the beam which occurs at high intensity ($\gtrsim 10^{12}$ ppp). Because of their limited frequency range response, these systems have the capability to act only on modes which group several bunches ($\gtrsim 10$) together, and have been fully effective for intensities up to $5\text{-}6 \times 10^{12}$ ppp.

At higher intensity ($\gtrsim 8 \times 10^{12}$ ppp) new modes of instability have been observed also with the dampers on. Isolated bunches, distributed in an erratic pattern, perform coherent vertical oscillations with growing amplitude. The growth stops when the beam size also gets large enough. This occurs around the transition energy and at higher energy also causing some problems with the extraction. It is expected that at higher intensity (we still have to go to 5×10^{13} ppp) the growth of the beam will be larger and, likely, losses of particles will occur, too.

The beam can be made stable with a fast damper capable of damping each bunch individually, but also by introducing enough tune spread. Because the instability is enhanced by oscillators (the particles) which are coupled to each other through a medium (the wake field), it is possible to lossen the coupling between them by having them oscillating at slightly different frequencies.

Likely, this is what is happening at the moment in the main ring. The beam is stable at low energy, because there is enough tune spread. At higher energy the average quality of the guide field improves and the beam size is reduced; thus, relatively, the spread decreases; the particles are now tightly coupled and resonate together. As a consequence, the beam oscillates as a whole at the collective frequency, which is always the betatron frequency, and the beam size also increases because of the smearing due to the spread, no matter how small. Once the beam gets bigger, the spread increases again, because particles have now larger oscillation amplitude and pick up more nonlinearities of the guide field. An equilibrium is then established between the beam size and the tune spread necessary to make the beam stable.

Bearing in mind the instability has to be cured also for the design intensity, the question arises: which method to use? A new fast feedback system or a large tune spread in the beam? Also, the instability might be sensitive on some parameter, like the chromaticity of the machine, easier to control. Finally, because the instability is caused by items surrounding the beam, one can try to modify the surrounding by localizing the dangerous items and replace them with modifications. We can have an answer for this question only if we have clear in our mind the details of the mechanism which makes the beam unstable.

The Ingredients

Before we look at the theories we should first list the main parameters that play a role in the instability.

(a) The charge density. Erroneously, often the intensity is taken as parameter. Actually, the size of the beam is also of primary importance. Obviously, we may expect that the growth rate of the instability increases linearly with the charge per unit volume and not just with the intensity.

(b) The energy. It is known that for relativistic particles the self-field has two contributions, one which is strongly energy dependent (usually like γ^{-2}) and depends only on the geometry of the beam and of the inner side of the vacuum tank, and a second one which is more familiarly known as "beam loading" and has no dependence at all with the energy. The second contribution depends on the electromagnetic properties of the surrounding. Thus the energy of the beam is important to determine the magnitude of the field as well as the balance between the two contributions.

(c) Beam configuration. By this we do not mean only the geometry of the beam but also, for example, whether the particles are performing synchrotron oscillations. The relative position of the particles obviously is important to determine in which way the particles interact with each other, namely which is the source of the perturbation field and which is affected by this. Also, the mutual position will change with time. There are only three possible configurations: (i) Coasting and uniform beam where particles have a velocity spread but do not perform synchrotron oscillations. The equilibrium density is constant and the same at any place, so that all the particles experience the same field no matter where they are located and no matter what their velocity is. (ii) Bunched beam. The particles now are performing synchrotron oscillations and experience a field of which the amount and phase depend on their location in the bunch. Since their location changes with time during one phase oscillation the self-field will also change accordingly. (iii) Chopped coasting beam. Although this seems unrealistic, nevertheless, it is a good approximation of a bunched be when the instability growth time is considerably smaller than a period of a phase oscillation. Also here, of course, the amount of the self-field depends on the position of the particle within the bunch, but, in some approximation, it is unchanged.

It is not difficult to convince ourselves that in the first and third configurations, the particles can be considered as one-dimensional oscillators, whereas in the second configuration we cannot disregard the fact that the particles have two modes of oscillations which are coupled through the chromaticity of the machine, thus we may expect that this parameter plays a crucial role in the instability.

(d) The wake field. We can expect different kinds of interactions between particles according to the decay and magnitude of the wake field. (i) When the wake field is large enough and decays very fast, say over the length of one bunch, interaction is possible only between particles of the same bunch or of the same chopped section of the beam. In the case of a bunched beam, there cannot be interaction between particles of different bunches. But in the case of coasting beam, because of the continuous charge distribution, there is always at least indirect interaction between any two parts of the beam. (ii) The wake field still decays over a short range but large enough to include several bunches, although not all of them. Although the interaction between particles of the same bunch can be still predominant, nevertheless also bunches next to each other interact. Provided the wake field does not have very accidental behavior, it is reasonable to disregard the internal motion in the bunch-bunch interaction, so that bunches look to each other as rigid. (iii) Finally the wake field decays over a large range which includes several revolutions. Here we have a multiturn effect: particles affect their own motion turn after turn. In this case also a small amount of wake field, properly in phase with the collective motion, can make the beam unstable.

(e) Tune spread. As we said before, this measures the coupling between the oscillating particles. There are two different ways to enhance a tune spread in the beam. (i) The size of the beam combined to the nonlinearities of guide field. This spread is a function of the energy because the size changes with the energy and the quality of the field depends on the excitation, namely on the energy too. Typically this spread decreases roughly linearly with the energy. (ii) The momentum spread of the beam times the chromaticity of the machine. For the same identical reasons, also this contribution to the tune spread is energy dependent and, likely, in the same way too. To understand the different role of these contributions, we better clear up the idea of tune spread needed to make the beam stable.[6] What is important is the separation in frequency between the oscillation of an actual particle and the collective oscillation during all the time the instability may take to develop. The tune spread gives the frequency distance averaged over the particle distribution. In the case the synchrotron motion is irrelevant, like in the case of coasting either uniform chopped beam, the frequency distance between the oscillations of two particles is indeed given by the combination of the two contributions mentioned above. Nevertheless, in the case the synchrotron motion is present and the instability develops over a period of time longer than the period of a phase oscillation, the frequency distance between two particles at long time is given only from the contribution of the amplitude of the betatron oscillations. Indeed the contribution from momentum deviation times chromaticity has zero average over a full momentum oscillation.

The Theoretical Models

We know of four theories which, with different degrees of approximation and for different situations, explain the transverse collective instabilities of an intense charged beam in a particle accelerator.

(a) Transverse Resistive Instabilities of Intense Coasting Beams in Particle Accelerators[1]

As the title says, this theory applies to a coasting (no synchrotron oscillations) and uniform beams. The wake field is created by the resistivity of the vacuum chamber and decays then, over a long range (several revolutions). This is a typical multiturn effect. Nevertheless the theory is easily generalized to any kind of field. The tune spread can come from the amplitude of the betatron oscillations as well as from the momentum spread. Indeed what is required is a spread in the quantity

$$S = (n-\nu)\Omega$$

where n is the unstable mode $(n>\nu)$, ν the betatron number and Ω is the angular velocity. Differentiating gives

$$\Delta S = (n-\nu)\Delta\Omega - \Omega\Delta\nu$$

Ω and ν are, obviously functions of the square of the beam size a^2 and of the momentum spread $\Delta p/p$. Thus we can write

$$\Delta S = \left[(n-\nu)\frac{\partial\Omega}{\partial\Delta p/p} - \Omega\frac{\partial\nu}{\partial\Delta p/p}\right]\frac{\Delta p}{p} \;+$$

$$+ \left[(n-\nu)\frac{\partial\Omega}{\partial a^2} - \Omega\frac{\partial\nu}{\partial a^2}\right] a^2 \;.$$

Usually $\frac{\partial\Omega}{\partial a^2} = 0$, and $\frac{\partial\Omega}{\partial\Delta p/p}$ is related to the momentum compaction factor, $\partial\nu/\partial\Delta p/p$ is the chromaticity, and $\partial\nu/\partial a^2$ the octupole component of the guide field. Observe that the signs of these quantities is important, for some combinations of them make the various contributions cancel each other.

Obviously this theory has no application to the main ring.

(b) Transverse Coherent Resistive Instabilities of Azimuthally Bunched Beams in Particle Accelerators[2]

This theory applies to a bunched beam. The wake field is assumed to decay over a long range so that bunches interact with each other and with themselves over several turns. The bunches are assumed to be rigid, namely the internal motion is ignored. In the case the times involved are larger than the synchrotron oscillation period, only the tune spread contributed by the beam size has a stabilizing effect. A combination of this contribution and the contribution of the momentum spread makes the beam stable when the times involved in the process are shorter than the synchrotron oscillation period. Observe that the contribution to the total tune spread is not necessarily additive.

Finally, whereas the single frequency response is only required in the coasting beam theory, here the integrated form of the field enters the calculation.

(c) Bunched Beam Breakup Theory to Explain Instabilities in CEA[3]

In this model bunches interact with each other by means of a wake field which nevertheless decays over a length shorter than one period of the machine. Thus, if the beam fills only a fraction of the accelerator circumference it is possible to have a situation where the bunch at the head of the beam is stable and the instability develops as we move to the bunches at the tail of the beam. How many bunches are affected by the instability depends, of course, on the amplitude, spatial frequency and decay of the wake field. Here, too, a tune spread makes a beam stable, and again the contribution from the momentum deviation is important only if the instability develops in a time considerably smaller than the period of a phase oscillation.

(d) Head-Tail Effect[4,5]

This model finally applies to the case the wake field is at such a short range that only particles of the same bunch interact with each other. There is the assumption that the instability develops over a period of time larger than the period of the synchrotron oscillation. Thus in this model the beam bunch is considered as a system of a large number of two-dimensional oscillators (longitudinal and transverse). The coupling between the two modes is obviously established via chromaticity. An elementary result is that by making the chromaticity identically zero the coupling between the two modes vanishes and the beam is stable.

Also, it is known that one sign (not the magnitude) of the chromaticity makes some transverse modes of oscillation stable and others unstable. Anyway with some chromaticity the beam bunch can be made stable introducing a tune spread via beam size. It has been found that in a proton synchrotron, usually the head-tail effect is more pronounced in proximity of the transition energy where the dependence of the revolution frequency with the energy vanishes.

Instabilities in the Main Ring[7]

(i) The phase oscillations period far away from transition is about 1 or 2 milliseconds.

(ii) Measurement of tune versus amplitude at 8 GeV and at low intensity (~ 10^{12} ppp) gave a tune spread of about 0.02 in either direction. The correcting octupoles can at most introduce a tune spread of (±) 0.01. At higher energy the spread can be several orders of magnitude smaller.

(iii) At low intensity, bunch-to-bunch traveling modes of the form $\exp i[n\theta-\omega t]$ with $\omega = (n-\nu)\Omega$ and $n>\nu$ have been observed. These modes are made stable either with octupoles (at rather low intensity) or with the dampers. This is a typical resistive wall bunched beam instability and the observations are in agreement with the theory.[8]

(iv) At moderate intensities a breakdown instability has also been found when only very few booster batches where injected. The head of the beam looks quite stable and a coherent instability develops from the head to the tail of the beam. Sometimes a gap of several bunches is artificially introduced in the string of batches to investigate the interaction length between bunches.

It was found that interaction occurs over only no more than 10 consecutive bunches. This, of course, is an indication of short range wake field which decays over a distance including several bunches but smaller than the machine circumference.

(v) The beam is usually stable at injection and unstable at high energy.

(vi) The beam can be made stable by turning on the correcting octupoles provided that the intensity is not too high otherwise the octupoles have no effect.

(vii) It seems the beam has a tendency to prefer positive chromaticity.

(viii) As it was said at the beginning of this paper, the actual dampers generally make the beam stable except at the very high intensity when isolated bunches, distributed in a random pattern, are unstable and their motion cannot be controlled.

(xi) The chromaticity can be made zero at injection but at high energy is negative in both planes.

One may conclude from these observations that we have a hierarchy of different kinds of instabilities in the main ring. If we assume, as it is reasonable, that the intensity threshold of each instability is correlated to the decay of the wake field, we can understand the order in which the various instabilities are showing up. We may expect also that at higher intensity the instability caused by wake fields with very short decay will predominate and will cause the instability mode which goes under the name of "head-tail" effect.

A possible item which produces a very short range wake field is a pair of pickup electrodes when terminated at their characteristic impedance. From the theory[9] we know that a particle which crosses such a device leaves a wake field proportional to the displacement y of the particle, of constant amplitude and extending over a length equal to twice the length of the plates. The force per unit of charge on a subsequent particle averaged over one turn is

$$F = \frac{4Me}{\pi^3 R} \frac{\beta \sin^2\delta}{b^2 C} y \tag{1}$$

$$= S m_o \gamma\, y$$

where

- M, number of pickup stations = 200
- e, particle charge
- R, machine radius = 10^5 cm
- β, ~ 1
- δ, half angular aperture of a plate = 90°
- C, capacitance per unit length between a plate and ground = 3
- ℓ, plate length = 13 cm
- $m_o\gamma$, relativistic mass of a particle

Eq. (1) applies only over a distance of 2ℓ = 26 cm, which likely is less than half of the bunch length.

The analysis of the head-tail effect enhanced by such a wake field has already been done[5]. In absence of a tune spread, the growth rate of the instability is given by

$$\beta_\mu = -\frac{2}{\pi^2} \frac{NS\xi A}{\alpha(4\mu^2-1)} \frac{2\ell}{A} \tag{2}$$

where S is obtained from (1) and

N, number of particles per bunch = 10^{10}

(corresponding to 10^{13} ppp)

$\xi = \Delta\nu/\nu/\Delta p/p$ chromaticity ~ - 1.1 (at high energy)

A = half bunch length in unit of time = 2×10^{-9}s

$\alpha = 1/\gamma_t^2 - 1/\gamma^2 \sim 0.003$ at high energy ($\gamma >> \gamma_t$)

μ = mode of instability, $\mu = 0$ involves oscillations of the centre of mass, and $\mu>0$ oscillations within the bunch which leave the centre of mass unchanged.

The rule is that if β_μ is positive the beam is stable, otherwise it is unstable. Since $S>0$, $\xi<0$ and α switches sign from negative to positive crossing the transition, we see that the fundamental mode $\mu = 0$ is unstable above transition and the higher modes, $\mu>0$, unstable below the transition but with a much smaller growth rate. Observe also that at transition $\alpha = 0$ and the growth rate is very large for any mode. Fortunately, the beam crosses the transition with some speed and we can expect just a growth in the beam at the present intensity and maybe we shall lose some beam at the design intensity. This is just what happened in the booster before sextupoles were installed to cancel the chromaticity of the machine. In that case, the wake field is likely cuased by the magnet laminations.

Inserting numbers in (1) and (2), we find that, at 100 GeV, the growth time of the $\mu = 0$ mode is of about 80 msec, for the present intensity of 10^{13} ppp.

At the design intensity (5×10^{13} ppp) and again at 100 GeV the growth time is of about 14 msec. Below the transition the most dangerous mode is of course $\mu = 1$ with growth time, at the design intensity, of 13 msec at 8 GeV.

The beam can be made permanently stable by setting the chromaticity to zero between injection and transition, which can be easily done and letting the chromaticity to the natural value (negative) above the transition, so that only the mode $\mu = 0$ is unstable, and drive the beam with a very fast damper capable of following each bunch individually.

Let us now look at the minimum tune spread required to make the beam stable. For this purpose one has to solve the following dispersion relation

$$-1 = i\beta_\mu \int \frac{f(a^2)\, da^2}{ka^2 - \omega_\mu} \tag{3}$$

where β_μ is given by (2) and (1) combined, $f(a^2)$ is the particle distribution in the square of the amplitude of the betatron oscillations, normalized to unity, ω_μ is the angular collective frequency minus the angular betatron frequency $\nu\Omega$, and $k = \Omega\, \partial\nu/\partial a^2$ is related to the strength of the average octupole. The solution of (3) is tedious and lengthy and depends strongly on the form chosen for $f(a^2)$. If we take

$$f(a^2) = \frac{1}{\langle a \rangle^2} e^{-\frac{a^2}{\langle a \rangle^2}}$$

where $\langle a \rangle$ is somewhat the measure of the beam radius we have as condition of stability[10]

$$\frac{|\beta_\mu|}{|k|\langle a \rangle^2} \lesssim .46$$

The amount of tune spread needed is

$$\Delta\nu = \frac{|k|\langle a\rangle^2}{\Omega} = \frac{|\beta_\mu|}{0.46\ \Omega}\ .$$

The results are shown in the following table where $\Delta\nu$ is tabulated for few cases.

μ	Energy	$1\text{x}10^{13}$ ppp	$5\text{x}10^{13}$ ppp
0	100 GeV	$8.6\text{x}10^{-5}$	$4.3\text{x}10^{-4}$
1	8 GeV	$5.1\text{x}10^{-4}$	$2.6\text{x}10^{-3}$

Well above transition, the amount of octupole needed increases linearly with the energy, because both β_μ and $\langle a\rangle^2$ go like $1/\gamma$. Well below transition, on the other hand, β_μ goes like γ and the octupole needed increases with the cube of the energy. Far away from transition it is not difficult to introduce the tune spreads listed above. For example, ~ 10 octupoles similar to the one used for extraction, located at F32, would be adequate for the design intensity up to 400 GeV. Also the actual system of correcting octupoles would be adequate below transition.

We do not know the amount of tune spread required at transition, but here, probably, it is better to cancel the chromaticity with sextupoles in the same way as it is done in the booster.

References

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6. See, for example, H.G. Hereward: "The Elementary Theory of Landau Damping", CERN 65-20, (May, 1965)
7. Mostly private communication from Rae Stiening. See also his paper presented to the 1974 Stanford International Conference on the High Energy Particle Accelerators.
8. A.G. Ruggiero, NAL Internal Note FN-217 (Oct. 1970)
9. See, for example, A.G. Ruggiero, Internal Note CRISP 73-21 (May, 1973). This is a summary of the theory
10. As we said the calculation is rather lengthy. We have in our file a memo with the details of the calculation.